A Simple Planning Problem for COVID-19 Lockdown *

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March 24, 2020

PRELIMINARY DRAFT

Abstract

We present a simple optimal control model for the COVID19 epidemic. We use the SIR epidemiology model described in Atkeson (2020) and Neumeyer (2020) and analyze an optimal lockdown policy. The planner's objective is to minimize the present discounted value of an objective function that depends on the fatalities and the output costs of the lockdown policy. We use some micro data to calibrate the key model parameters. The quantitative result are useful to gauge what parameters of the problem are important in shaping the intensity and duration of the optimal lockdown policy. Our preliminary results, conditional on an initial 1% fraction of infected agents and no cure for the disease, prescribe a total economic lockdown for about 5 weeks, followed by a fast relaxation of the lockdown policy, which however persists for a long period of time. The intensity of the lockdown depends critically on the gradient of the fatality rate as a function of the infected.

*First draft, March 23th 2020. We are circulating this very preliminary version for discussion among colleagues. We will keep updating it, and check for its accuracy.

1 Introduction

We present a simple optimal control model for the COVID19 epidemic. We adopt a variation in the SIR epidemiology model reviewed and proposed by Atkeson (2020) and analyze an optimal lockdown policy. Our aim is to contribute to the ongoing discussion on the optimal policy response to the COVID shock, see Barro, Ursua, and Weng (2020); Eichenbaum, Rebelo, and Trabandtz (2020); Gourinchas (2020); Dewatripont et al. (2020) and the contributions in the volume by Baldwin and Weder (2020).

The typical approach in the epidemiology literature is to study the dynamics of the pandemic, for infected, deaths, recovered, as functions of some exogenously chosen diffusion parameters β, which are in turn related to various policies (e.g. the partial lock down of schools, universities, offices, and other measures of diffusion mitigation), and where the diffusion parameters are stratified by age and other individual covariates. This is the approach followed for instance by Ferguson and Et.al (2020). We use a simplified version of these models developed by Atkeson (2020) to analyze how to balance the fatality induced by the epidemic with the output costs of the lockdown policy. The novel aspect of our analysis is to explicitly formulate and solve a control problem where the diffusion rate β is chosen to maximize a given social objective and taking into account the dynamic nature of the problem. An optimal control problem based on a very similar epidemiological model can be found in Hansen and Troy (2011), but the objective function and the feasible policies are different.

The planner's objective is to minimize the presented discounted value of fatalities while trying to minimize the output costs of the lockdown policy, since agents in lockdown are assumed not to produce. Underlying our model is a SIR epidemiology model, describing how the virus propagates from the Infected agents to those Susceptible of falling ill, as well as the rates at which, once infected, agents either recover or die. In particular, we assume that the fatality rate (probability of dying conditional on being infected) is state dependent, i.e. that it increases once the hospital capacity is reached. Such a non-linear behavior gives the planner a motive to smooth the curve of the infected, a central argument in most analyses of this problem. We explicitly formulate and solve the problem of a planner who in the short run has access to a single instrument to deal with the epidemic: the lockdown of the citizens.[1] Needless to say the analysis has several limitations: the underlying model has no heterogeneity in fatality rates nor in diffusion rates, the lockdown policy is imperfect (some contact rate remains even in a full lockdown) but cannot be differentiated across agent's type (e.g. young versus old, workers vs retirees). We also completely ignore direct health interventions that might be put in place to mitigate the consequences of the disease (building emergency hospitals, extensive testing and tracing, incentivizing the research and cure for the disease). Our objective is to put this problem into a broader picture by combining the dramatic health hazards of the COVID with the equally dramatic economic consequences of an extreme reduction of economic activity.

The quantitative result are useful to gauge what parameters of the problem are important in shaping the intensity and duration of the optimal lockdown policy. Several comparative statics analysis are developed. Our preliminary benchmark results, conditional on an initial 1% fraction of infected agents and no cure for the disease, prescribe a total economic lockdown for about 5 weeks, followed by a fast relaxation of the lockdown policy, which however persists for a long period of time. After 1 year, half of the economic activity is still in a lockdown. The intensity of the lockdown depends critically on the gradient of the fatality rate as a function of the infected. As we flatten this function the length of the lockdown converges to zero. We explore the importance of the effectiveness of the lockdown.

Our objective is similar to that pursued by Eichenbaum, Rebelo, and Trabandtz (2020): while they focus on a a competitive equilibrium where a consumption tax is used to slow-down economic activity as well as the epidemic diffusion, we focus on a simpler policy that allows the planner to control the level of citizens in a lockdown. One difference is that while their problem is convex, and thus generally delivers an internal solution (a tax rate in between zero

[1] While this is not the only margin of action (other actions might involve reinforcing health treatment capacity and incentivizing the development of vaccines), in the short run this seems to be an important policy tool available and used by several countries.

and infinity), our problem is non-concave in large regions of the state space, where corner solutions (no lockdown or complete lockdown) become optimal.

The paper is organized as follows: the next section describes the planner's problem and the epidemic model. Section 3 uses some micro data to calibrate the key model parameters drawing from Atkeson (2020); Ferguson and Et.al (2020). Section 4 discusses the preliminary results of the optimal control problem under different scenarios.

2 A planner model of lockdown control

We start with a modified version of the by now well known SIR model as described in Atkeson (2020). Agents can be divided between those susceptible to be infected $S(t)$, those infected $I(t)$, and those recovered $R(t)$, i.e.

$$N(t) = S(t) + I(t) + R(t) \text{ for all } t \geq 0 \tag{1}$$

Those "recovered" include those that have been infected and are now assumed to be immune and those have died. We normalize the population to $N(0) = 1$. The planner can decide to lock down a fraction $L(t) \in [0, \bar{L}]$ of those susceptible and those infected, where $\bar{L} \leq 1$ allows us to realistically consider that even in a disaster scenario some economic activity such as energy and basic food production have to continue. Recovered agents are not in lockdown. A fraction of those that are in lockdown, cannot be infected, nor can infect some other susceptible agents. We assume that the lockdown is only partially effective in eliminating the transmission of the virus. When L agents are in lockdown, then $(1 - \theta L)$ agents can transmit the virus, where $\theta \in (0, 1]$ is a measure of the lockdown effectiveness. If $\theta = 1$ the policy is fully effective in curbing the diffusion, but since some contacts will still happen in the population even under a full economic lockdown we allow $\theta < 1$.

The law of motion of the susceptible agents is:

$$\dot{S}(t) = -\beta\, S(t)(1 - \theta L(t))\, I(t)(1 - \theta L(t)) \tag{2}$$

where β is the number of susceptible agents per unit of time to whom an infected agent can transmit the virus, among those not in lockdown. All susceptible agents that get the virus become infected. For the infected, a fraction γ recovers, thus:

$$\dot{I}(t) = \beta\, S(t)(1 - \theta L(t))\, I(t)(1 - \theta L(t)) - \gamma I(t) \tag{3}$$

A rate $0 < \gamma_d(I) \leq \gamma$ per unit of time of those infected die. Thus population decreases due to death among those infected as:

$$-\dot{N}(t) = D(t) = \gamma_d(I(t))\, I(t) \tag{4}$$

While we assume that the rate γ at which infected recover is exogenous, the fraction of those recovering that die is assumed to depend on the number of infected I. This completes the description of the epidemiological block of the model.

We assume that each alive agent produces w units of output, when she is not in lockdown. Agents are assumed to live forever, unless they die from the infection. The planner discounts all values at the rate $r > 0$. We assume that with probability ν per unit of time both a vaccine and a cure simultaneously appear, so that all infected are cured and all susceptible become immune, ie. all infected and susceptible become recovered. The planner seeks to maximize:

$$\int_0^\infty e^{-(r+\nu)t} \Big((N(t) - [S(t) + I(t)]\, L(t))\, w + \dot{N}(t)\chi + \frac{\nu}{r} N(t) w \Big)\, dt \tag{5}$$

by choice of the control $L(t)$ at all times, i.e. choosing fraction of agents to lock down $L(t)$. The objective function includes the present value of the output of those alive, as well as an extra cost χ, in units of output, for each agent that dies as a consequence of the virus, above and beyond the loss output. The term $\frac{\nu}{r}N(t)w$ is the product of the probability that a cure takes place ν, times the discounted value of output from there on, $wN(t)/r$. The planner problem is subject to the law of motion of the susceptible individuals equation (2), the infected equation (3), the population, equation (4), and an initial condition $(N(0), I(0), S(0))$ with $I(0) > 0$ and $S(0) + I(0) \geq N(0)$.

2.1 Discussion of modeling assumptions

We now briefly discuss the key modeling assumptions

1. If $\theta = 1$ the lockdown is able to completely stop the infections process, i.e. to achieve $\dot{S} = 0$. If $\theta < 1$ the effectiveness of the lockdown policy is partial (people keep transmitting the virus) but at a lower rate.

2. We restrict the extent of the lockdown to $\bar{L} \leq 1$. This is to take into account, in a very rough manner, that some sectors cannot shut down even with the most severe lockdown, such as heath, basic services, food production and distribution, etc.

3. Those recovered are assumed to be immune, and we assume that are not locked down. It seems like a property of any optimal policy, provided previously infected agents cannot contract the infection again (see Dewatripont et al. (2020) for a related policy).

4. In the law of motion for S and I given by equation (2) and equation (3) we do not scale by the population $N(t)$ to compute the number of infected at time t. Instead we simply write, $\beta I(t)S(t)(1 - \theta L(t))^2$, instead of $\beta I(t)S(t)(1 - \theta L(t))^2/N(t)$. This seems to be the standard in the SIR literature, although it will be preferable to scale them by $N(t)$. Since some agents die, this cannot be done by simple setting $N(0) = 1$. We follow the literature, since this also save in one state variable.

5. We do not include an intermediate state $E(t)$, as it is done in some of the SIR models

–see Atkeson's note–, where agents have no symptoms but can transmit the virus. This may be interesting to include.

6. We assume that agents that are infected but not in locked down, can still produce as much as those susceptible or recovered not in locked down. This can be easily changed, but a better model will have two type of infected, one with symptoms and one without. Those with symptoms should be locked-down first, and only after then, one should isolate the rest.

7. The assumption that $\gamma_d(I)$ is increasing in I, that is postulated in equation (8), tries to capture that as I increases, the capacity to treat patients diminishes and then rate at which infected agent dies increases.

8. We assume that all agent are infinitely lived, except for the risk of dying after contracting the virus. This simplification is acceptable given the short span horizon of the problem. On the other hand, not having an explicit age structure is very unrealistic in the impact of mortality risk.

9. The planning problem incorporates two type of costs. One is the economic activity lost during a lockdown among those susceptible and infected, and also due to the death of infected agents. Note that since we use infinitely lived agent, the death of an infected person implies a cost equal to the present value of this person income, which overestimates the economic cost of death, measured as the statical value of life. We also include an extra cost of each death, χ.

10. We can use a negative value of $\chi < 0$, with the interpretation that the statistical like of those that die as a consequence of the virus is smaller, due to shorter remaining life-span, and also if one does not want to set a value of life beyond the remaining discounted earnings.

An equivalent problem for the planner that conserves in the state space is given by minimizing the present discounted value, discounted at rate $r+\nu$, of the following flow cost:

$$wL(S+I) + I\gamma_d(I)\left[\frac{w}{r}+\chi\right] \tag{6}$$

so the cost of having a state (S,I) and a current control L is that a fraction L of output of those susceptible and infected $(S+I)w$. For those that die as a consequence of the infection, we subtract the present value of output that they would have produced, w/r, as well as the extra cost of death, χ. This leaves the problem as one with two states, (S,I).

The planner solves the following Bellman-Hamilton-Jacobi equation:

$$\begin{aligned}(r+\nu)V(S,I) = \min_{L\in[0,\bar{L}]} & \; wL(S+I) + I\gamma_d(I)\left[\frac{w}{r}+\chi\right] + \\ & - \left[\beta\, S(t)(1-\theta L(t))\, I(t)(1-\theta L(t))\right]\partial_S V(S,I) \\ & + \left[\beta\, S(t)(1-\theta L(t))\, I(t)(1-\theta L(t)) - \gamma I(t)\right]\partial_I V(S,I)\end{aligned} \tag{7}$$

The domain of V is $(S,I)\in\mathbb{R}^2$ such that $S+I\leq 1$. Note that $V(S,I)$ can be interpreted as the minimum expected discounted cost of following the optimal policy, in units of output loss. Recall that if $\gamma_d=0$, then the objective of the planner at time $t=0$ will give $N(0)w$. Thus, the quantity $rV(S,I)/w$ converts the stock-value of the value function into a ratio of the flow cost relative to output at time $t=0$, when there is no virus.

We solve this problem by discretizing the model to daily intervals, using value function iteration in a grid, combined with interpolation outside grid points to evaluate the value function outside.

3 A preliminary parametrization of the model

We calibrate our model using data from the World Health Organization compiled by the Johns Hopkins University Center for Systems Science and Engineering (JHU CCSE) while acknowledging, like the rest of the recent literature, that at this point there is considerable uncertainty about infection, recovery and mortality rates. The data includes the total cases, including separately those that have recovered and those that have died. We define active cases as the total number of cases minus those that either recovered or died. We use daily observations of all the countries that have registered at least 100 active cases and include observations of the first 25 days after they first cross this threshold. The data used here was last updated on March 20, 2020.

To calibrate β, the rate at which individuals who are infected bump into other people and shed virus onto those people, we use the daily increase in active cases and assume a value of 20 percent. Figure 8 shows the total number of active cases by the number of days since the 100th case. The black dashed line represents a daily increase of 33 percent and the red dashed line a daily increase of 20 percent. The figure shows that $\beta = 0.2$ is a lower bound for this parameter. The parameter γ governing the rate (per day) at which infected people either recover or die is considered a fixed parameter of the disease and is set to γ=1/20 reflecting an estimated duration of illness of 20 days as in Atkeson (2020) but also consistent with the fraction of infected agents that recovered or died depicted in Figure 9. The figure shows the total number of cases that either recover or die and the total active cases. The fitted line shows the fraction of infected agents that recover or die after assuming that this fraction is 5 percent (i.e. γ=1/20).

The per day rate at which infected people die, which we refer to as the fatality rate, is assumed to have the following functional form

$$\gamma_d(I) = \gamma_d + \kappa I \tag{8}$$

which reflects that the capacity to treat patients diminishes and, as a result, the rate at which infected agents die increases. We choose γ_d to be consistent with the mortality rates observed in the data. Figure 10 shows the number of new deaths and the total active cases. The fitted line shows the fraction of infected agents that died after assuming that this fraction is 1 percent (i.e. γ_1=0.01). Since our model is daily, we set γ_d=0.01 $\times$ γ. We set κ=0.1

$\times\ \gamma$ so that the peak of mortality rate is 5 percent. There is considerable uncertainty on the mortality rate, mostly because the true rate of infected is not really know. For instance, Eran, Jay, and Sood (2020) argue that the number of infected is probably at least an order of magnitude larger, and thus the mortality rate much smaller.

We set the planner's discount factor to be consistent with a 5 percent annual interest rate and the per unit of time probability ν that a vaccine and a cure will appear so that it implies that it takes on average a year and a half for these medical discoveries to become available. We normalize output w=1 and set the extra cost of dying χ to be equal to 10% of the present value of output. Lastly, we assume that even in a disaster scenario economic sectors such as health, government, retail, utilities, and food manufacturing will continue. These sectors combined account for 25-30% of GDP (2018). Thus, we set $\bar{L} = 0.75$.

Table 1: Parameter Values and Data Moments

Note: The table indicates the parameters values used in the calibration of the model and the data moments used to choose them. The daily increase in active cases, the fraction of active cases that recover or die (per day), and the new deaths per active case (per day) are estimated using data from the World Health Organization. The data was last updated on March 20, 2020. The share of GDP of health, retail, government, and utilities is from are based on the 2018 estimates by the Bureau of Economic Analysis (BEA).

Parameter	Value	Moment
β	0.2	Daily increase active cases.
γ	1/20	Fraction of active cases that recover or die (per day).
γ_d	$0.01 \times \gamma$	New deaths per active case (per day).
κ	$0.1 \times \gamma$	Peak of mortality rate at 5 percent.
r	0.05	Annual interest rate.
ν	0.0018	Probability per day for medical discoveries to appear (1.5 years)
$\bar{L}$	0.75	1 minus the share of GPD of health, retail, government, utilities.
w	1	Normalization.
χ	0	Normalization.

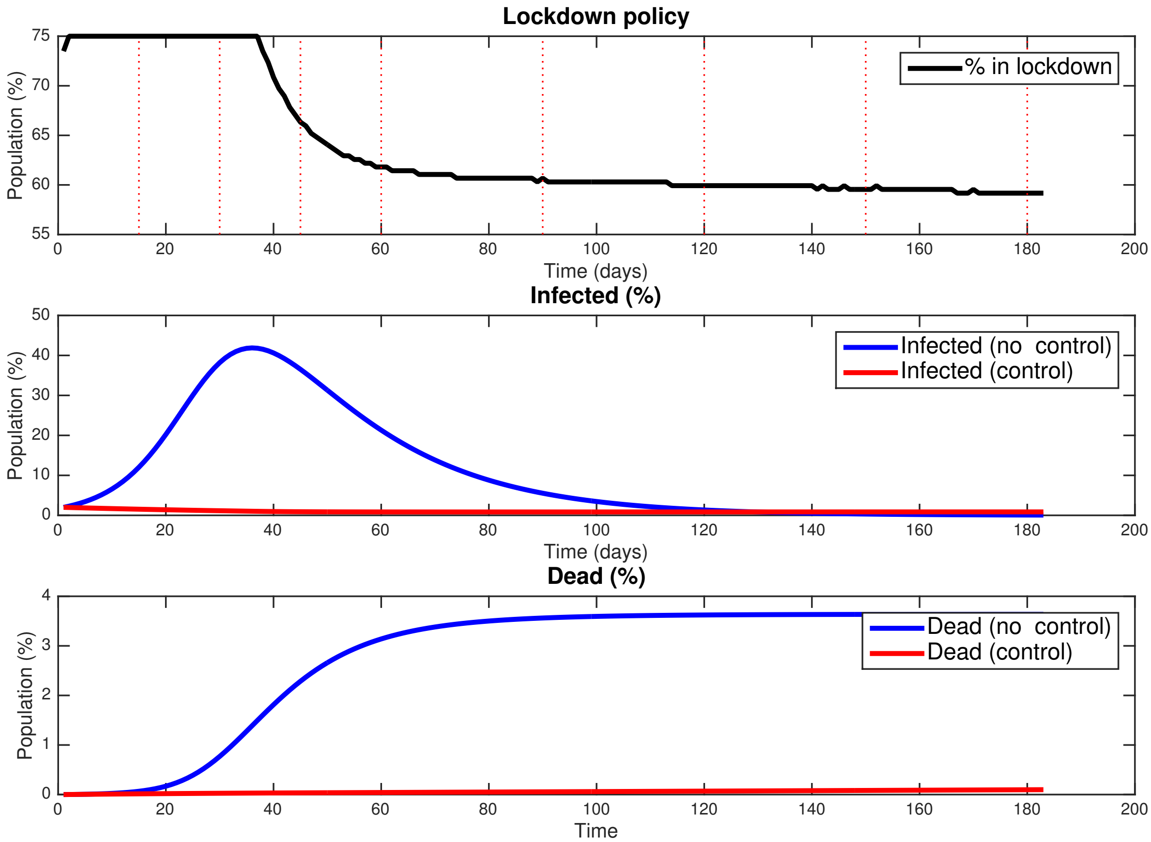
It goes without saying that the values for several parameter are speculative. We will conduct some sensitivity analysis to try to illustrate their importance.

4 Results

We display the time path of the optimal policy starting at $I(0) = 0.01$, i.e. one percent of population infected at $t = 0$ for our benchmark parameter values. We display the time path of the optimal lockdown policy $L(t)$ as function of time, the path of infected $I(t)$, and the total accumulated fraction of dead. Recall that $N(0) = 1$, so both infected and the stock of dead can be all interpreted as fraction of the initial population. The horizontal axis is time, measured in days, conditional on the cure-vaccine not occurring before that period. For comparison, we also plot the path if there is no policy, i.e. for $L(t) = 0$ for all $t \geq 0$.

In what follow we first present the result for our benchmark parameter case, in Figure 1 and Figure 2, as well as its associated value function and optimal policy Figure 5. For this parameters, an initial total lock-down, which then is decreases slowly is optimal, which implies an extreme flattening of the curve of infected. We explore the sensitivity to parameter values by changing the effectiveness of the lockdown, i.e by reducing it from $\theta = 0.8$ to $\theta = 0.5$ in Figure 3 or even to $\theta = 0.25$ in Figure 4. In these cases of less effective lockdown, it is optimal to start with a lockdown, but the curve is flattened much less. Figure 7 has the interesting case of $\kappa = 0$, so that the mortality rate does not increase with the number of infected. In this case the result change dramatically, and it is optimal have zero lockdown, and this remains true even if we increase the baseline fatality rate, while keeping $\kappa = 0$. We discuss each of this cases in more detail below.

Figure 1: Short run: High lockdown effectiveness ($\theta = 0.8$)



We display these time series with two different time scale, in Figure 1 and Figure 2. These two panels correspond to a value of the effectiveness of the lockdown of $\theta = 0.8$, which we deem as reasonable. The optimal policy if a complete lock-down that last for almost a month and a half, and a gradual elimination of the lockdown, for a very protracted period of time –see Figure 2, only to be stopped after more than three year. Recall that all times are measured, conditional on no cure-vaccine available. Note that the lockdown policy is very effective in reducing the number of infected, and hence the number of deaths, with an extreme flatting of the curve of infected.

Figure 2: High lockdown effectiveness ($\theta = 0.8$)

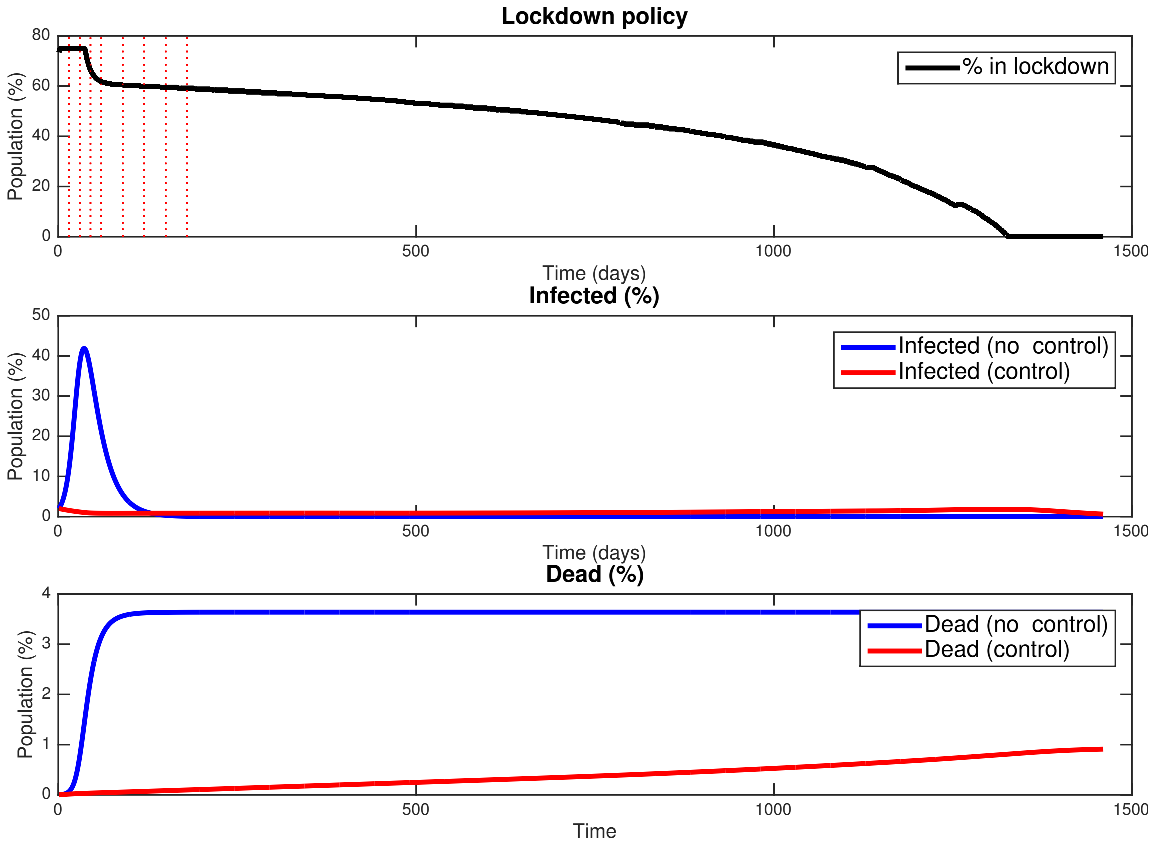
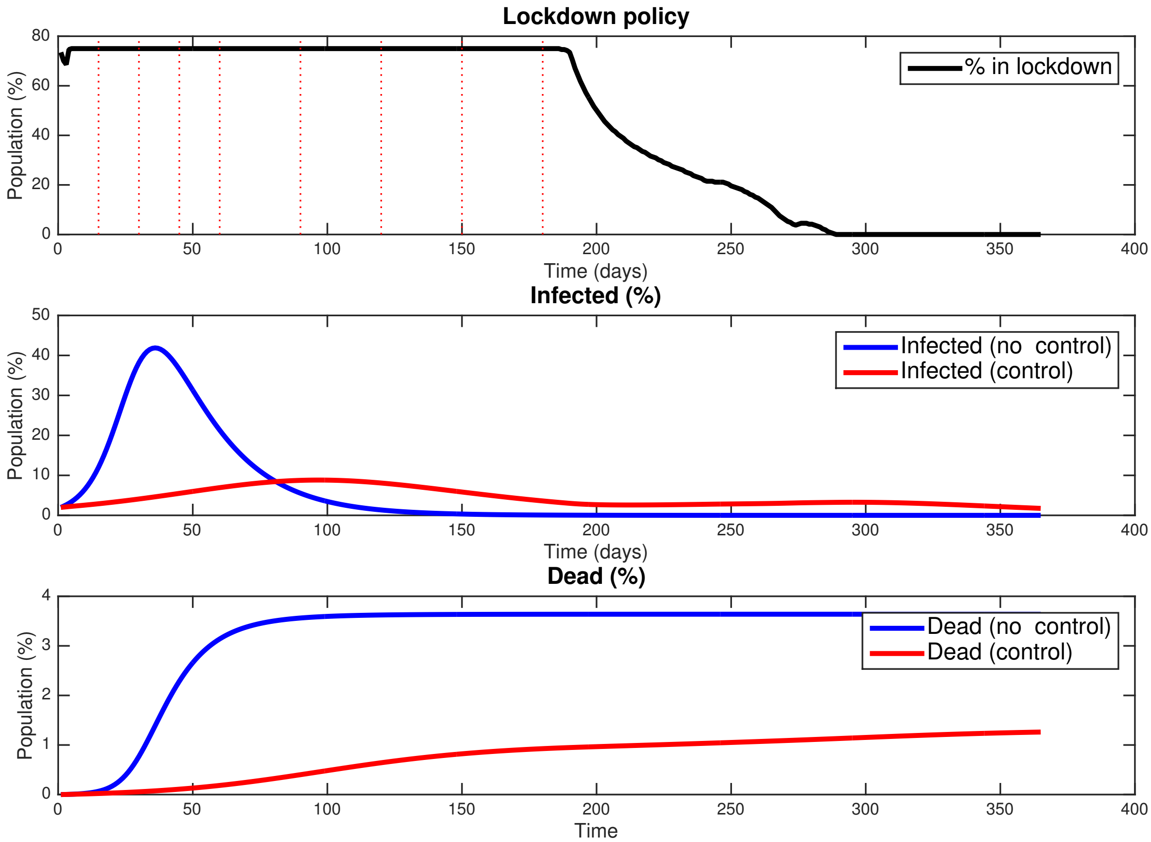


Figure 3: Medium lockdown effectiveness ($\theta = 0.5$)



For comparison, Figure 3, reduces the effectiveness of the lockdown to $\theta = 0.5$. In this case, the total lockdown last longer, but it is also reduced and abandoned faster. The curve of infected is flattened, but peaks much earlier than in the $\theta = 0.8$ case. Figure 4 has the case of an even less effective lockdown technology, with $\theta = 0.25$. In this case the optimal policy is a shorter total lockdown, lasting less than two months, and eliminated almost immediately after. The curve of infected is only partially flattened here.

Figure 4: Low lockdown effectiveness ($\theta = 0.25$)

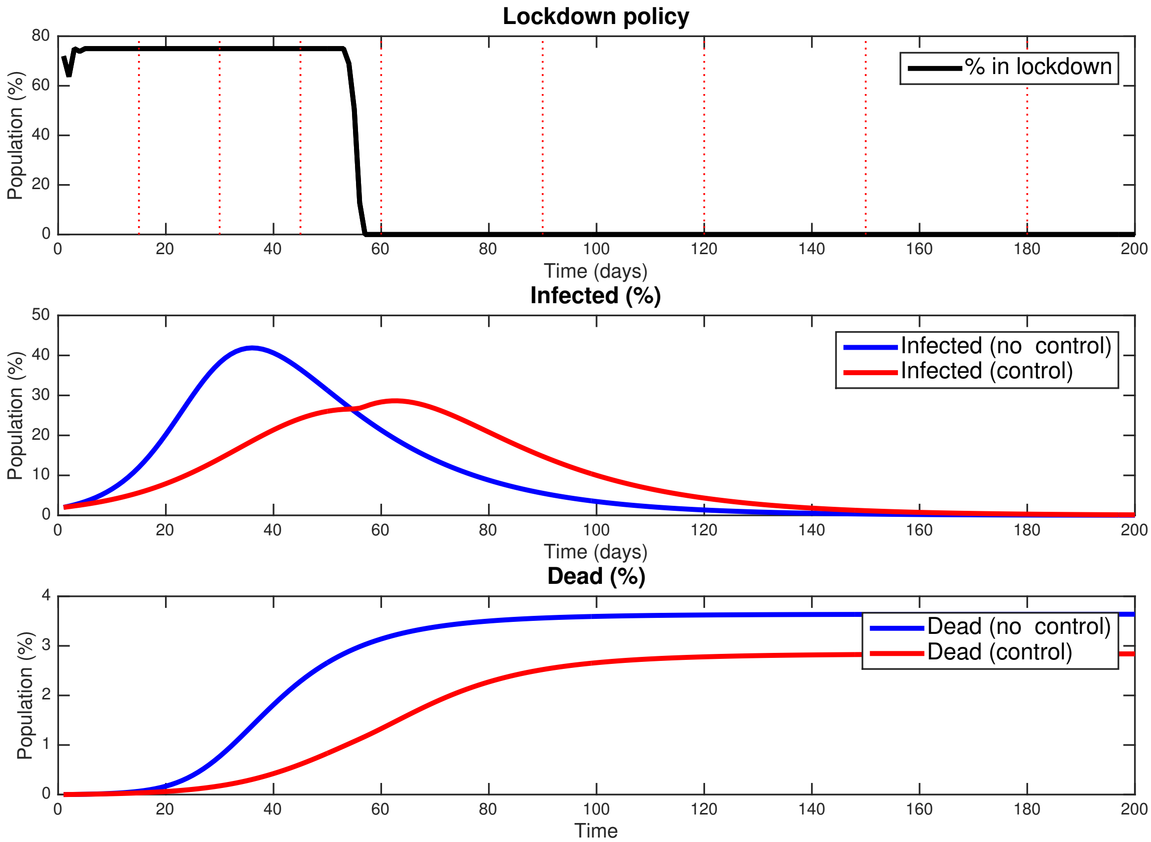


Figure 5 displays the value function and the optimal policy for the benchmark parameter values. The value function is plotted in the right panel, for the relevant state space (S, I), and normalized as explained above, so we display $rV(S, I)/w$ in the vertical axis. The units are thus, permanent flow cost as a fraction of the total output previous to the virus. Thus, a value of 0.05, means a cost equivalent to a permanent reduction of 5% percent in the value of output. The left panel panel has a heat map of the optimal policy $L^*(S, I)$. Darker-blue color signify no lock down, i.e. $L = 0$, and lighter-yellow color indicate $L^*(S, I) = 1$.

Figure 5: Value Function and Optimal Policy ($\theta = 0.8$)

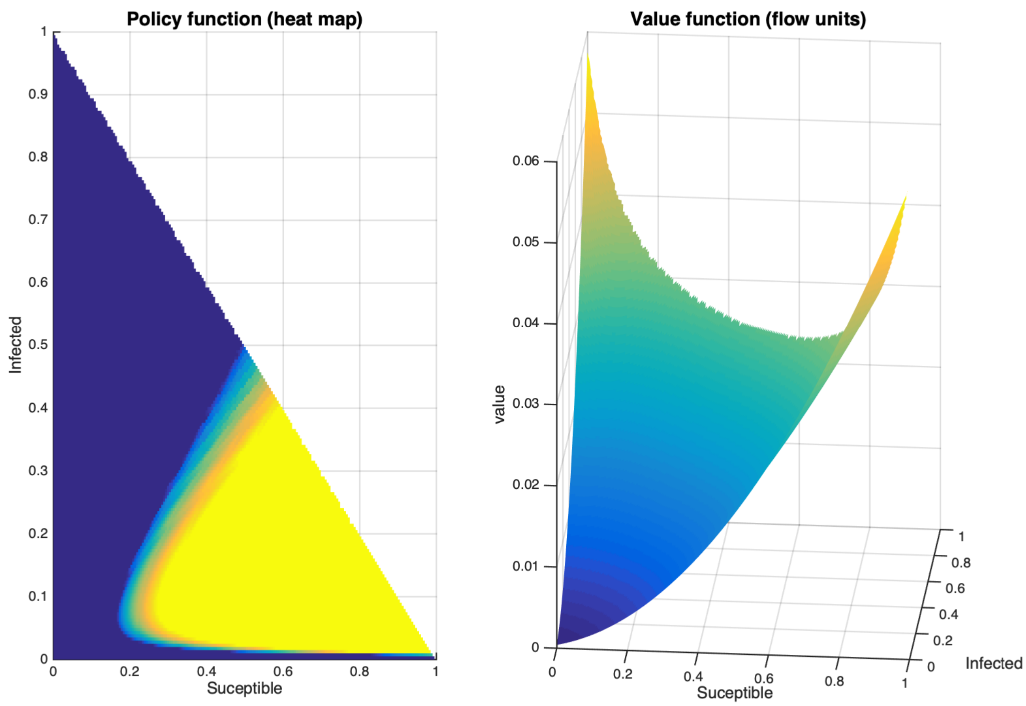
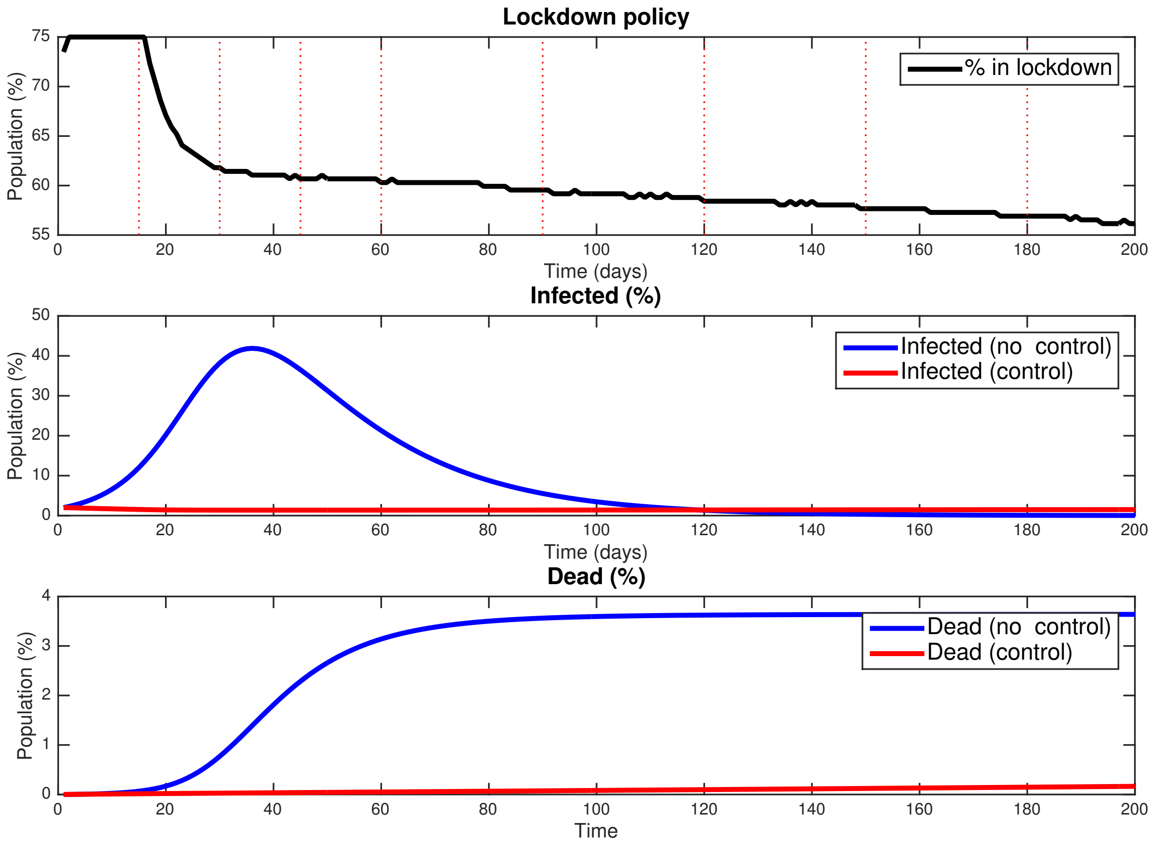


Figure 6 explores the case in which $\chi < 0$. This is done, as explained above, to understand the case in which the planner puts no weight on the death agents beyond its contribution to earnings, and where those that die have lower statistical value of life. This can, in a simplified way, capture that the fatality rate of the virus is concentrated among those with fewer remaining work-years. In particular we assume that χ is such that those that die as a consequence of the virus, have half of the expected future life-time earnings. The outcome is intuitive, with the lower economic cost of fatalities, the optimal lockdown is shorter. Nevertheless, even with this large departure, the qualitative nature of the optimal policy is similar.

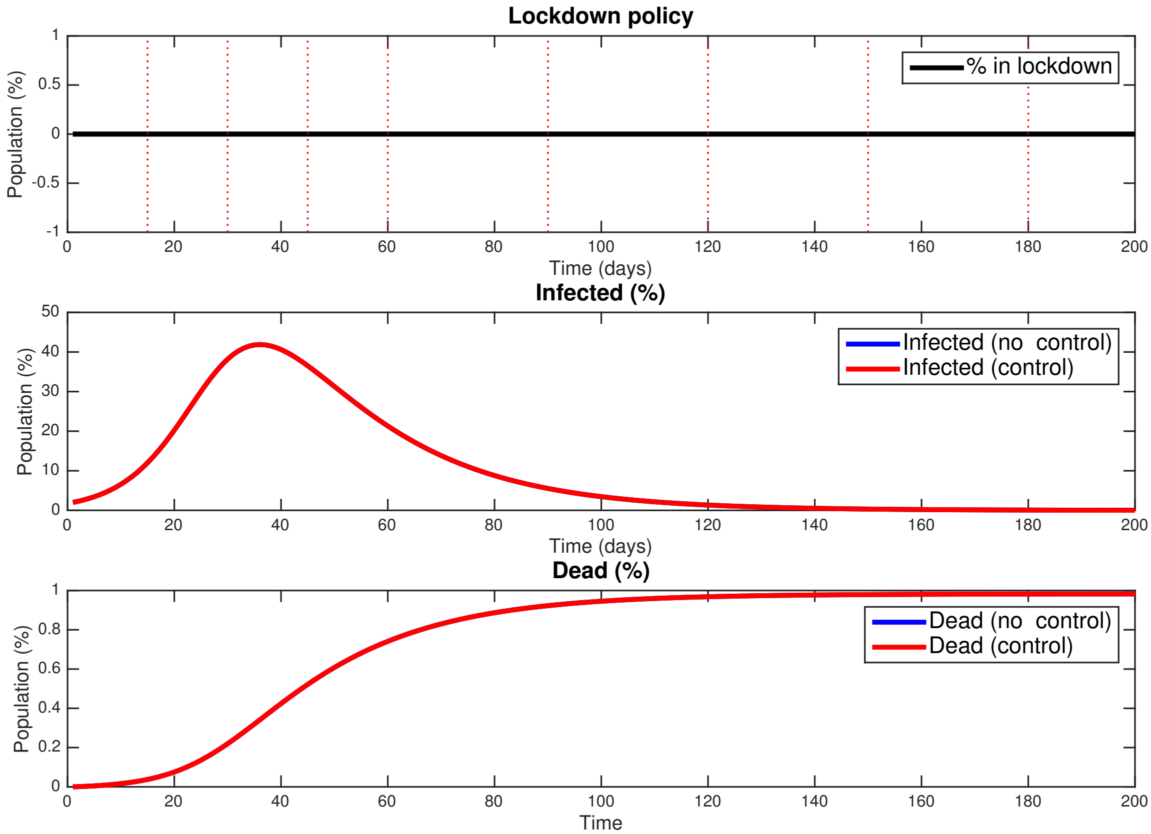
Figure 6: High effectiveness ($\theta = 0.80$) with $\chi = -\frac{1}{2}\frac{w}{r}$



Inspecting the role of the fatality rate. The behavior of the fatality rate, described in equation (8), is critical in determining the optimal lockdown policy.

Figure 7 shows the result of assuming that $\gamma_d(I)$, the rate at which infected agents die is independent of the number of infected patients in the population, i.e. we assume that $D(t) = -\dot{N}(t) = \gamma_d I(t)$ or equivalently that $\kappa = 0$. We keep all the other parameters at the benchmark values. In this case the optimal policy changes dramatically, and it is to have nobody lockdown, i.e. $L(t) = 0$ for all $t \geq 0$, for any value of (I, S). This result is not trivial, since having $L(t) > 0$ reduces the long run number of deaths. Interestingly, keeping $\kappa = 0$, even if we increase the baseline death rate to $\gamma_d = 0.2$ or to even $\gamma_d = 0.5$, the optimal policy is still $L(t) = 0$. We conclude from this exercise that the sensitivity of the death rate to the level of infected patients is very important to determine the optima lockdown.

Figure 7: High effectiveness ($\theta = 0.80$) with constant fatality rate $\kappa = 0$



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5 Appendix

5.1 The optimization problem

The Hamiltonian for this problem is:

$$H(S,I,L,\lambda,\mu) = w(S+I)L + I\gamma_d(I)\left(\frac{w}{r}+\chi\right) + \lambda\dot{S} + \mu\dot{I} \tag{9}$$

$$\dot{S} = -\beta SI(1-\theta L)^2 \tag{10}$$

$$\dot{I} = \beta SI(1-\theta L)^2 - \gamma I \tag{11}$$

The first order conditions are, like in any problem: $\partial_L H = 0$, $\dot{\lambda} = (r+\nu)\lambda - \partial_S H$ and $\dot{\mu} = (r+\nu)\mu - \partial_I H$. Writing the Hamiltonian explicitly we have:

$$H(S,I,L,\lambda,\mu) = w(S+I)L + I\gamma_d(I)\left(\frac{w}{r}+\chi\right) - \lambda\beta SI(1-\theta L)^2 + \mu(\beta SI(1-\theta L)^2 - \gamma I)$$

and thus

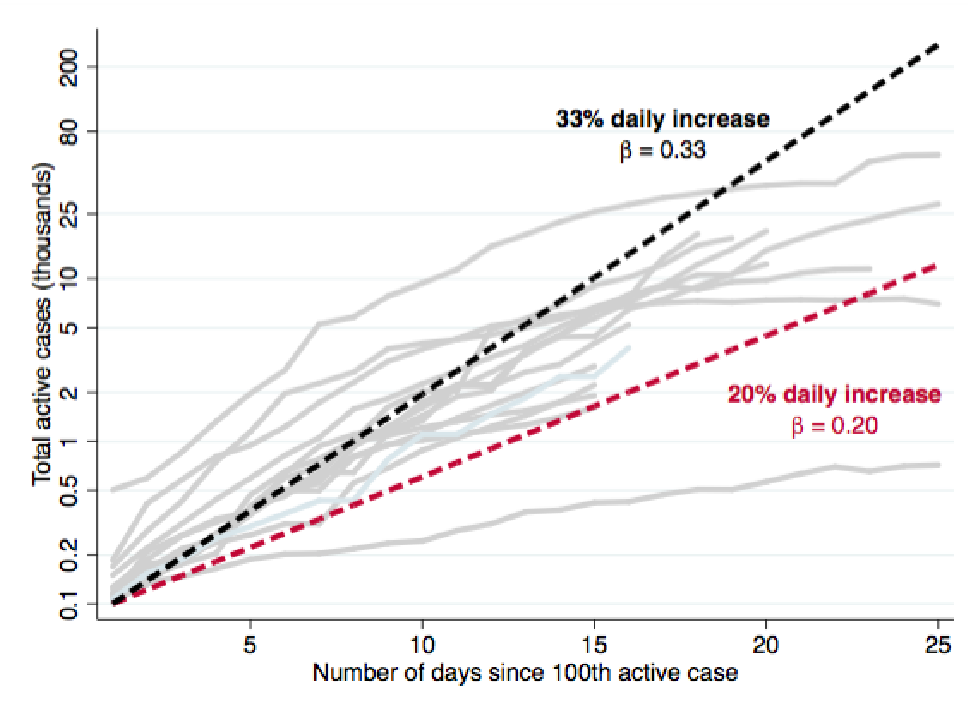
$$\begin{aligned}
\dot{\lambda} &= (r+\nu)\lambda - wL + (\lambda-\mu)\beta I(1-\theta L)^2 \\
\dot{\mu} &= (r+c+\gamma)\mu - wL - (\gamma_d(I) + I\gamma_d'(I))\left(\frac{w}{r}+\chi\right) + (\lambda-\mu)\beta S(1-\theta L)^2 \\
\partial_L H &= w(S+I) + (\lambda-\mu)\beta SI2(1-\theta L)\theta = 0
\end{aligned}$$

Note that

$$\dot{\lambda} - \dot{\mu} = (r+c)(\lambda-\mu) + \mu\gamma + (\lambda-\mu)\beta(1-\theta L)^2(I-S) + (\gamma_d(I) + I\gamma_d'(I))\left(\frac{w}{r}+\chi\right)$$

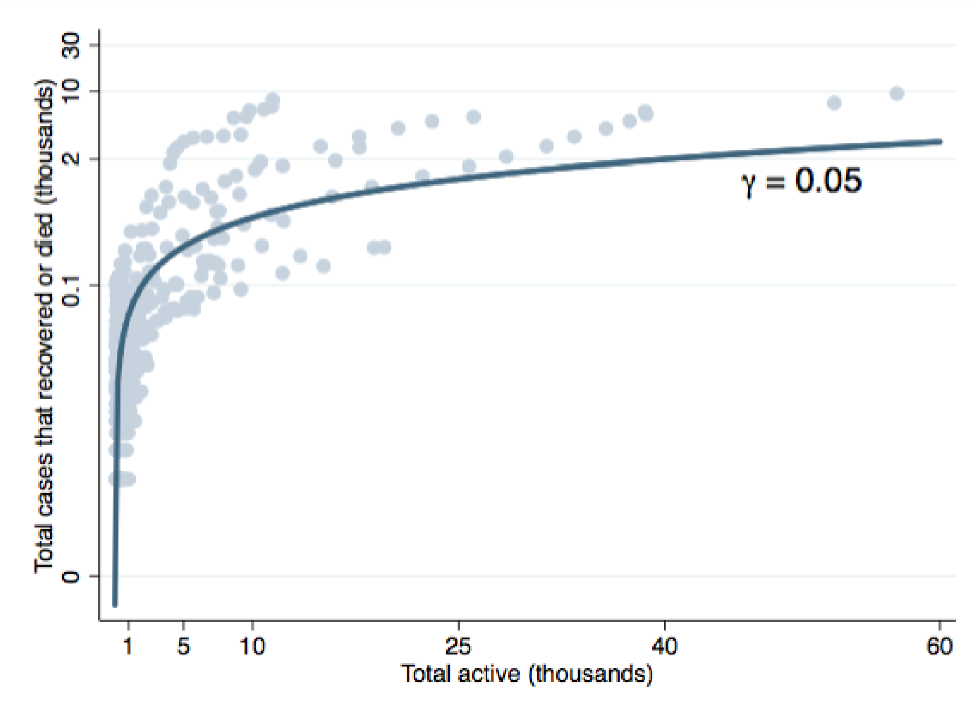
5.2 Empirical facts behind parametrization

Figure 8: Daily Increase in Active Cases



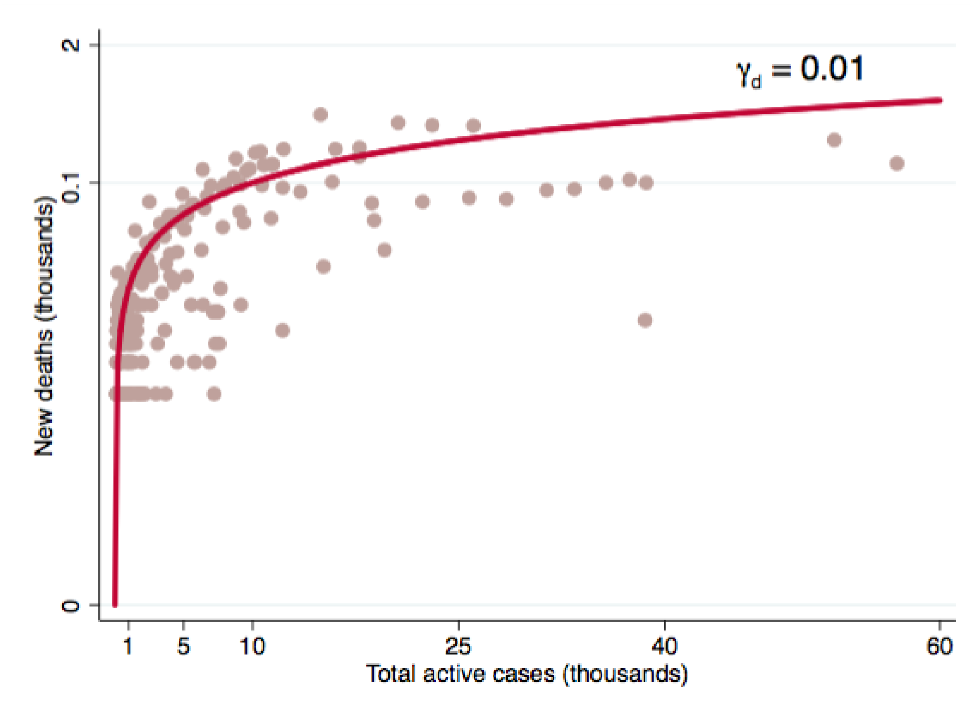
Note: The figure shows the total number of active cases by the number of days since the 100th case. The number of active cases is the cumulative number of cases minus the cases that either recovered or died. The black dashed line represents a daily increase of 33% and the red dashed line a daily increase of 20%. The data include daily observations of all the countries that have registered at least 100 active cases. We include observations of the first 25 days after crossing this threshold. The source is data from the World Health Organization (WHO) compiled by the Johns Hopkins University Center for Systems Science and Engineering (JHU CCSE). The data set was last updated on March 20, 2020.

Figure 9: Fraction of Infected Agents that Recovered or Died



Note: The figure shows the total number of cases that either recovered or died and the total active cases. The number of active cases is the cumulative number of cases minus the cases that either recovered or died. The fitted line shows the fraction of infected agents that recovered or died after assuming that this fraction is 5% (i.e. γ=0.05). The data include daily observations of all the countries that have registered at least 100 active cases. We include observations of the first 25 days after crossing this threshold. The source is data from the World Health Organization (WHO) compiled by the Johns Hopkins University Center for Systems Science and Engineering (JHU CCSE). The data set was last updated on March 20, 2020.

Figure 10: Fraction of Infected Agents that Died



Note: The figure shows the number of new deaths and the total active cases. The number of active cases is the cumulative number of cases minus the cases that either recovered or died. The fitted line shows the fraction of infected agents that died after assuming that this fraction is 1% (i.e. γ_1=0.01). The data include daily observations of all the countries that have registered at least 100 active cases. We include observations of the first 25 days after crossing this threshold and days in which at least one death is registered. The source is data from the World Health Organization (WHO) compiled by the Johns Hopkins University Center for Systems Science and Engineering (JHU CCSE). The data set was last updated on March 20, 2020.